

Geometry I, 2006 : Coursework 4

Show your calculations.

1. Let a, b, c, d, α be real numbers, with $\alpha \neq 0$.

(i) [2 marks] Prove that if $x = p, y = q, z = r$ is a solution of $ax + by + cz = d$, then it is also a solution of $\alpha ax + \alpha by + \alpha cz = \alpha d$.

(ii) [2 marks] Prove also that if $x = p, y = q, z = r$ is a solution of $\alpha ax + \alpha by + \alpha cz = \alpha d$, then it is also a solution of $ax + by + cz = d$.

2. Let $a, b, c, d, e, f, g, h, \alpha$ be real numbers, and consider the following systems of linear equations:

$$(*) \quad \begin{cases} ax & + & by & + & cz & = & d \\ ex & + & fy & + & gz & = & h \end{cases};$$

$$(**) \quad \begin{cases} ax & + & by & + & cz & = & d \\ (e + \alpha a)x & + & (f + \alpha b)y & + & (g + \alpha c)z & = & h + \alpha d \end{cases}.$$

(i) [2 marks] Prove that if $x = p, y = q, z = r$ is a solution of $(*)$ then it is also a solution of $(**)$.

(ii) [2 marks] Prove also that if $x = p, y = q, z = r$ is a solution of $(**)$ then it is also a solution of $(*)$.

3. [6 marks] For each of the following systems of linear equations in x, y, z , determine all the solutions:

$$(i) \quad \begin{cases} 3x & - & y & - & 2z & = & -11 \\ x & - & y & + & z & = & -6 \\ 5x & - & 3y & - & 4z & = & -23 \end{cases};$$

$$(ii) \quad \begin{cases} 3x & - & y & - & 2z & = & 11 \\ 6x & - & y & - & 4z & = & 22 \end{cases};$$

$$(iii) \quad \begin{cases} 243x & + & 53y & + & 163z & = & 912 \\ 1024x & + & 28y & + & 923z & = & 78 \\ 1024x & + & 28y & + & 923z & = & 79 \\ 847x & - & 33y & + & 7843z & = & -105 \end{cases}.$$

4. [6 marks] Let Π be the plane defined by $2x - 3y + 6z = 4$. For each of the three (non-degenerate) linear equations below, determine the intersection of Π with the plane defined by that equation. In each case, give the intersection as a set of points. If the intersection is a line or a plane then also give equation(s) defining this line or plane.

(i) $-4x + 6y - 12z = -8$; (ii) $-4x + 6y - 12z = 8$; (iii) $-4x + 6y - 11z = 8$.

Put your name and student number on your solutions, staple them, and put them in the **Geometry I** Collecting Box in the **Basement** of the Maths Building by 5pm on Thursday 26th October.

Leonard Soicher, 19 October 2006